

Antiferromagnetic spinor condensates in a two-dimensional optical lattice

L. Zhao, J. Jiang, T. Tang, M. Webb, and Y. Liu*

Department of Physics, Oklahoma State University, Stillwater, OK 74078

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We experimentally demonstrate that spin dynamics and the phase diagram of spinor condensates can be conveniently tuned by a two-dimensional optical lattice. Spin population oscillations and a lattice-tuned separatrix in phase space are observed in every lattice where a substantial superfluid fraction exists. In a sufficiently deep lattice, we observe a phase transition from a longitudinal polar phase to a broken-axisymmetry phase in steady states of lattice-confined spinor condensates. The steady states are found to depend sigmoidally on the lattice depth and exponentially on the magnetic field. We also introduce a phenomenological model that semi-quantitatively describes our data without adjustable parameters.

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A spinor Bose-Einstein condensate (BEC) confined in optical lattices has attracted much attention for its abilities to systematically study, verify, and optimize condensed matter models [1–3]. For instance, it can quantum simulate the Laughlin-type wavefunctions appearing in the fractional quantum Hall systems [4, 5]. A better understanding of these models may directly lead to engineering revolutionary materials. An optical lattice has been a versatile technique to enhance interatomic interactions and control the mobility of atoms [6–8]. Atoms held in a shallow lattice can tunnel freely among lattice sites and form a superfluid (SF) phase. The tunneling rate is exponentially suppressed while the on-site atom-atom interaction is increased in a deeper lattice. This may result in a transition from a SF phase to a Mott-insulator (MI) phase at a critical lattice depth, which has been confirmed in various scalar BEC systems [6–9]. In contrast to a scalar BEC, a spinor BEC has unique advantages due to an additional spin degree of freedom. The SF-MI phase transition is predicted to be remarkably different in spinor BECs, i.e., the transition may be first (or second) order around the tip of each Mott lobe for an even (or odd) occupation number in lattice-trapped antiferromagnetic spinor BECs [1, 10].

Spin-mixing dynamics and phase diagrams of spinor BECs in free space, as a result of spin-dependent interactions and quadratic Zeeman energy q_B , have been well studied with sodium atoms [11–17] and rubidium atoms [18–21]. Richer spin dynamics are predicted to exist in lattice-trapped spinor BECs, which allow for a number of immediate applications. These include constructing a novel quantum-phase-revival spectroscopy driven by a competition between spin-dependent and spin-independent interactions, understanding quantum magnetism, directly detecting spin-dependent three-body and higher-body interactions, and realizing massive entanglement [1, 3, 22]. However, dynamics of lattice-trapped spinor BECs have remained to be less explored, and most of such experimental studies have been carried out in ferromagnetic ^{87}Rb spinor BECs [23–26].

In this paper, we experimentally demonstrate that a two-dimensional (2D) optical lattice can conveniently tune spin dynamics and map the phase diagram of $F=1$ antiferromagnetic spinor BECs. We find that the properties of spinor BECs remain largely unchanged in the presence of a shallow lattice, while a sufficiently deep lattice introduces some interesting changes. First, in every lattice depth u_L which supports a substantial superfluid fraction, we observe spin population oscillations after taking spinor BECs out of equilibrium at a fixed q_B . These oscillations are resulted from coherent interconversion among two $|F=1, m_F=0\rangle$ atoms, one $|F=1, m_F=+1\rangle$ atom, and one $|F=1, m_F=-1\rangle$ atom. Second, we demonstrate a lattice-tuned separatrix in phase space and explain it using lattice-enhanced spin-dependent interactions. Another remarkable result is our observation of a phase transition from a longitudinal polar phase to a broken-axisymmetry (BA) phase in steady states of spinor BECs confined by sufficiently deep lattices. We find that the steady states depend sigmoidally on u_L and exponentially on q_B . We also introduce a phenomenological model that semi-quantitatively describes our experimental data without adjustable parameters. This model takes into account the observed time evolutions of quantum depletion, resulting mainly from the lattice-flatten dispersion relation.

We create a BEC of 7×10^4 sodium atoms fully polarized into the $|F=1, m_F=-1\rangle$ state in a crossed optical trap via an all-optical BEC method similar to that of our previous work [15]. To adiabatically load the BEC into a 2D lattice, we decompress the optical trap to a value which minimizes intra-band excitations and ensures approximately constant Thomas-Fermi radii during a linear ramping of the lattice potential within $t_{\text{ramp}} > 40$ ms. We construct the 2D lattice using two linearly-polarized horizontal beams which originate from a single-mode laser at $\lambda_L = 1064$ nm, have a waist of $\sim 90 \mu\text{m}$ at the condensate, and are retro-reflected to form standing waves. To eliminate cross interference between different beams, the two lattice beams are frequency-

shifted by 20 MHz with respect to each other. u_L are calibrated using Kapitza-Dirac diffraction patterns. Note that all lattice depths studied in this paper are kept below $15.0(5)E_R$ to avoid SF-MI phase transitions and thus maintain a sufficient superfluid fraction in our system. Here $E_R = \hbar^2 k_L^2 / (8\pi^2 M)$ is recoil energy, $k_L = 2\pi/\lambda_L$ is the lattice wave-number, M is the atomic mass, and \hbar is the Planck constant. We apply a resonant rf-pulse of a proper amplitude and duration to lattice-trapped BECs for preparing an initial state with any desired combination of the three m_F states at $q_B/\hbar = 42$ Hz. q_B is then quenched to a desired value within a wide range (i.e., $20 \text{ Hz} \leq q_B/\hbar \leq 1700 \text{ Hz}$). After holding atoms for various amounts of time t_{hold} , we abruptly switch off all lattice and trapping potentials, and then measure populations of multiple spin states with standard Stern-Gerlach absorption imaging.

In the presence of a shallow lattice of $u_L < 5E_R$, we observe spin population oscillations which are very similar to those occur in free space: the oscillations are harmonic except near a separatrix in phase space where the oscillation period diverges, as shown in Fig. 1. We define ρ_{m_F} as the fractional population of each m_F state. The total magnetization $m = \rho_{+1} - \rho_{-1}$ is found to be conserved in every time evolution studied in this paper. As the lattice is made deeper, the oscillations appear to damp out more quickly and the position of the separatrix in phase space shifts to a much higher q_B . Similar to Refs. [1, 3], we apply the Bose-Hubbard model to understand our system. There are three important terms in this model: the spin-dependent interaction energy U_2 , the spin-independent interaction U_0 , and the tunnelling energy J among adjacent lattice sites. U_2 is proportional to the atomic density in each lattice site, and is positive (or negative) in $F=1$ ^{23}Na (or ^{87}Rb) BECs. In fact, $U_2/U_0 \simeq 0.036$ for our ^{23}Na system [3]. For the initial state studied in Fig. 1, we find $U_2 \simeq q_B$ at each separatrix in phase space. The observed lattice-tuned separatrix in phase space (i.e., the separatrix position shifts with u_L) is thus mainly due to the fact that U_2 greatly increases with u_L . Fig. 1(b) shows a good numerical example: U_2/\hbar is more than doubled (increased from 14 Hz to 32 Hz) by changing u_L from $2.5E_R$ to $4.5E_R$. Interestingly, we find that our data taken at $u_L < 5E_R$ can also be fit by predictions derived from the single-spatial mode approximation (SMA), as shown in Fig. 1(b). SMA assumes that all spin states share the same spatial wavefunction [27]. Sharp interference peaks are observed after we release spinor BECs from a shallow lattice, which indicates coherence and superfluid behavior in the system. The inset in Fig. 1(b) shows a typical absorption image taken after a 5-ms time of flight (TOF).

Spin oscillations completely damp out and spinor BECs reach their steady states when t_{hold} is long enough, as shown in Fig. 2(a). Sufficiently deep lattices are found to bring some interesting changes to the steady states.

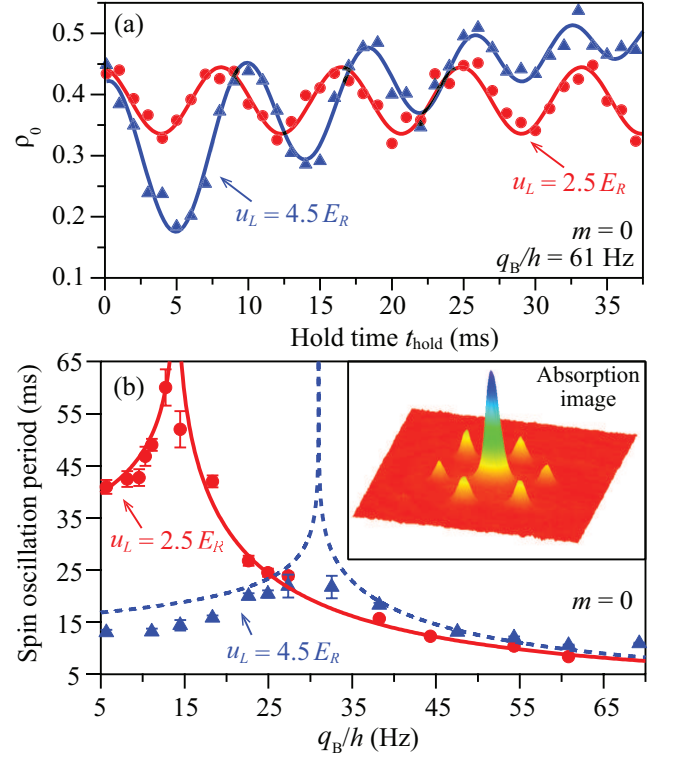


FIG. 1. (Color online) (a) Time evolutions of ρ_0 when u_L equals $4.5E_R$ (blue triangles) and $2.5E_R$ (red circles). Solid lines are sinusoidal fits to extract oscillation periods. (b) Oscillation period as a function of q_B when u_L equals $4.5E_R$ (triangles) and $2.5E_R$ (circles). Lines are fits based on SMA. Inset: an absorption image averaged from 30 raw images.

Figure 2(a) demonstrates one of such changes: once u_L is larger than a critical value, the steady states undergo a phase transition from a longitudinal polar phase (where $\rho_0 = 1$) to a BA phase (where $0 < \rho_0 < 1$) at $m = 0$. We repeat the same measurements with only one parameter changed, i.e., by blocking the retro-reflected path of each lattice beam. The two lattice beams are effectively independent, blocking their retro-reflected paths thus eliminates standing waves and constructs a crossed optical dipole trap (ODT). Its resulting trap depth is u_{ODT} , as illustrated in the inset in Fig. 2(b). Note that the power of every beam in Fig. 2(b) is four times of that in Fig. 2(a) to ensure $u_L = u_{\text{ODT}}$. Our data in Fig. 2(b) show that spinor BECs at $m = 0$ always reach the polar phase when there are no standing waves. The dramatically different results shown in Figs. 2(a) and 2(b) provide strong evidence of the necessity to understand this polar-BA phase transition with lattice-modified band structures.

We then study spin oscillations and steady states within a much wider range of u_L and m . Steady states appear to depend sigmoidally on u_L at a fixed q_B , as shown in Fig. 3(a). The inset in Fig. 3(a) demonstrates another surprising result: the observed relationship between ρ_0 and m in steady states at a sufficiently large u_L

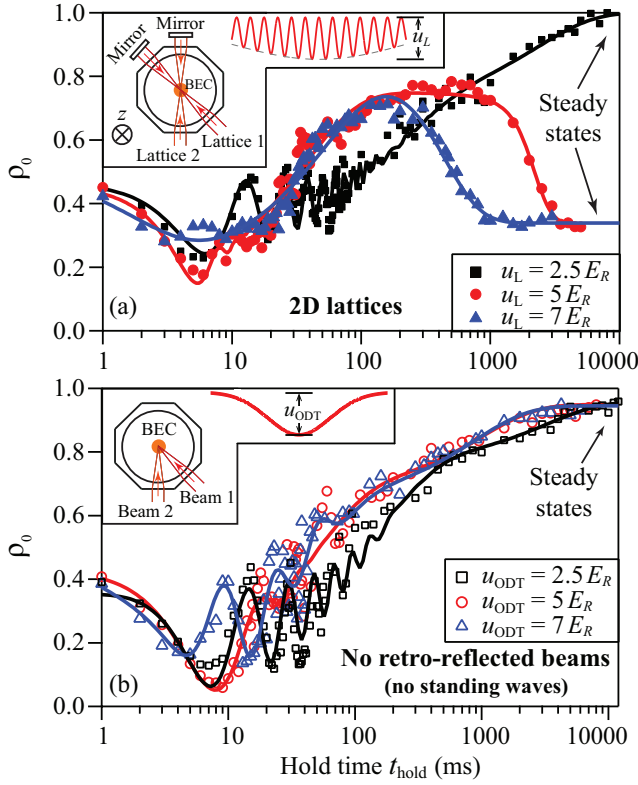


FIG. 2. (Color online) (a) Time evolution of ρ_0 at $q_B/h = 42$ Hz and $m = 0$ when u_L equals $2.5E_R$ (black squares), $5E_R$ (red circles), and $7E_R$ (blue triangles). Inset: a schematic of our lattice setup and an illustration of the resulting lattice potential. Lines are fits to guide the eye. (b) Similar to Panel (a) except that each beam is not retro-reflected.

can be well fit by $\rho_0 = (1 - |m|)/3$, which is drastically different from a well-known mean-field prediction (i.e., $\rho_0^{D=0}$ as illustrated by the blue dashed line) [28]. This mean-field prediction assumes quantum depletion D is zero, where D is defined as the fraction of atoms stay in non-zero momentum states. The $D \approx 0$ assumption is correct in free space and in very shallow lattices for our system, as predicted by Bogoliubov theory [9]. We extract D from TOF images, and confirm $D < 5\%$ at $u_L \leq 3E_R$. Note that the trapping frequency in each lattice site is much bigger than U_0/h [9]. Our TOF images thus reflect the momentum distribution at the instant of the lattice release and enable us to directly measure D .

We also find that D increases with t_{hold} and u_L , and approaches one in steady states of spinor BECs when $u_L > 10E_R$, as shown in Fig. 3(b). This lattice-enhanced quantum depletion is resulted mainly from the lattice-flatten dispersion relation and lattice-enhanced interactions, and was also observed in a scalar BEC system [9]. We develop one phenomenological model to take into account the observed D . Surprisingly, this model semi-quantitatively describes our data without adjustable parameters, as shown in Fig. 3(a) and 4. In this model, the

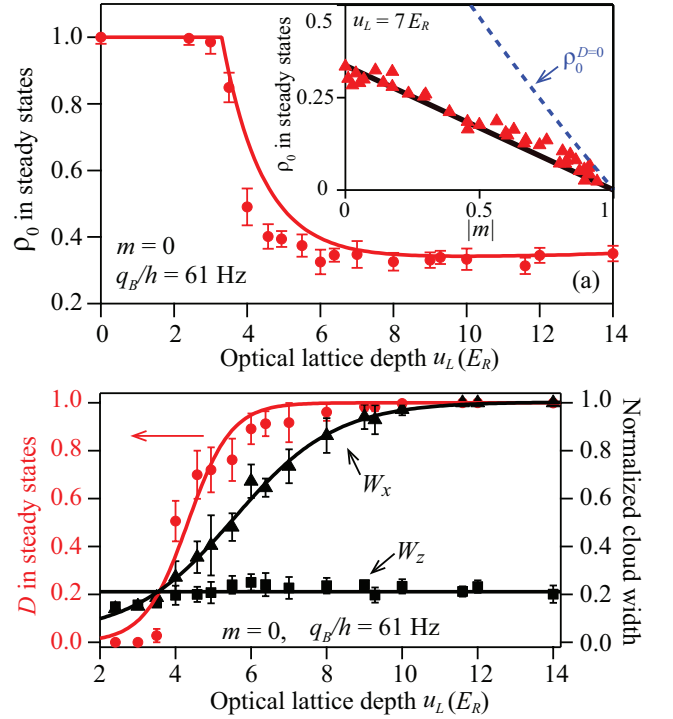


FIG. 3. (Color online) (a) ρ_0 in steady states as a function of u_L at $m = 0$ (main figure), and as a function of $|m|$ at $u_L = 7E_R$ (inset figure). Solid lines are predictions derived from Eq. (2). The blue dashed line represents $\rho_0^{D=0}$ (see Ref. [28]). (b) The values of W_x (black triangles), W_z (black squares), and D (red circles) in steady states as a function of u_L . The widths are normalized by k_L . Lines are fits to guide the eye.

steady states are determined by a comparison between $T(\mathbf{k}, m_F = 0)$ and $T(0, m_F = \pm 1)$, where $T(\mathbf{k}, m_F)$ is the dispersion relation of the m_F state and \mathbf{k} is the atom's quasi-momentum. The inset in Fig. 4 illustrates two such comparisons in a shallow lattice ($u_L = 3E_R$) and a deep lattice ($u_L = 10E_R$). Note that only the first Brillouin zone is considered, since the population in higher bands is negligible. Similar to Ref. [7–9], we calculate the dispersion relation of spinor BECs in a 2D lattice using a Wannier density function along each of the two horizontal directions with lattices (the x -axis and y -axis) and a uniform density function along the vertical direction without a lattice (the z -axis) as follows,

$$T(\mathbf{k}, m_F) = 4J \sum_{\alpha=x,y} \sin^2 \left(\frac{\pi k_\alpha}{2k_L} \right) + E_R \frac{k_z^2}{k_L^2} + q_B m_F^2. \quad (1)$$

Here the linear Zeeman effect is ignored because it remains the same in the coherent inter-conversions. $T(\mathbf{k}, m_F = \pm 1)$ is thus shifted up by q_B with respect to $T(\mathbf{k}, m_F = 0)$ at a fixed u_L and a given magnetic field. The inset in Fig. 4 shows that the dispersion relations are significantly flattened when u_L increases from $3E_R$ to $10E_R$. In fact, the predicted width of the first band is $\sim 4J$, where J exponentially reduces with u_L [8, 9].

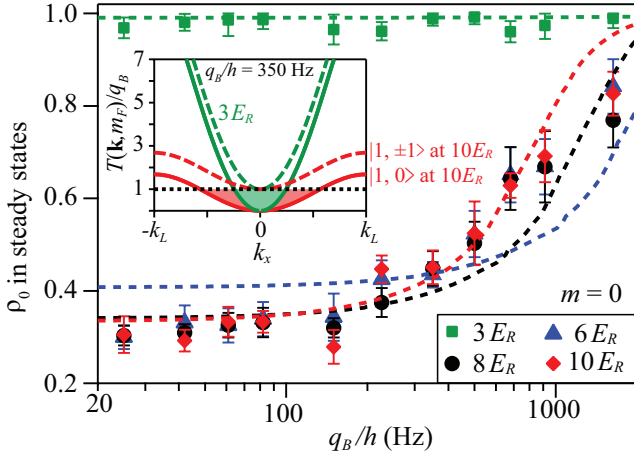


FIG. 4. (Color online) ρ_0 in steady states as a function of q_B at $m = 0$ when u_L equals $3E_R$ (squares), $6E_R$ (triangles), $8E_R$ (circles), and $10E_R$ (diamonds). Dashed lines are predictions derived from Eq. (2). Inset: the dispersion relations normalized by q_B as a function of k_x when $k_y = k_z = 0$. The solid (or dashed) lines represent results of the $m_F = 0$ (or $m_F = \pm 1$) states. The dotted line marks $T(\mathbf{k}, m_F)/q_B = 1$ and colored regions mark the Region-1 (see text). In the main and inset figures, the green, blue, black, and red colors respectively represent results at u_L being $3E_R$, $6E_R$, $8E_R$, and $10E_R$.

We divide $T(\mathbf{k}, m_F = 0)$ into two regions based on $T(0, m_F = \pm 1)$, i.e., the minimum energy of the $m_F = \pm 1$ states. Region-1 is the first region in which $T(\mathbf{k}, m_F = 0) < T(0, m_F = \pm 1)$, as illustrated by the colored regions in the inset in Fig. 4. To clearly explain our model using the dispersion relations shown in Fig. 4, we only consider $m = 0$ in this paragraph. In Region-1, atoms in the $m_F = 0$ state always have smaller energy than those in the $m_F = \pm 1$ states. The steady states should be the $m_F = 0$ state, i.e., $\rho_0 = 1$ which is identical to the mean-field prediction $\rho_0^{D=0}$. When quantum depletion D is big enough, atoms start to occupy Region-2 where $T(\mathbf{k}, m_F = 0) \geq T(0, m_F = \pm 1)$. In Region-2 the three m_F states have the same minimum energy, $T(0, m_F = \pm 1)$, atoms in steady states should thus evenly distributed among the three states. In other words, $\rho_0 = 1/3$ in Region-2, which is identical to the phenomenological relationship extracted from Fig. 3(a).

We can apply a similar discussion and our model to all non-zero m . Thus ρ_0 in the steady states is expressed as,

$$\rho_0 = \int_{\text{Region-1}} n(\mathbf{k}) \rho_0^{D=0} d\mathbf{k} + \int_{\text{Region-2}} n(\mathbf{k}) \frac{1 - |m|}{3} d\mathbf{k}. \quad (2)$$

Here $n(\mathbf{k})$, the normalized atomic density in steady states, is calculated as follows: $n(\mathbf{k}) = (1 - D)\delta_{\mathbf{k}} + D \exp[-(k_x^2/W_x^2 + k_y^2/W_y^2 + k_z^2/W_z^2)/2]$, where \mathbf{W} is the half-width of a Gaussian fit to a TOF distribution, $W_y = W_x$, and δ is a Dirac-delta function. Figure 3(b) shows that W_x and D sigmoidally increase with u_L , and

saturate at their peak values when $u_L > 10E_R$. In other words, atoms occupy all available states and quantum depletion saturates the first Brillouin zone in a deep lattice. In contrast, W_z appears to be independent of u_L , which implies the system temperature remains unchanged.

The results derived from our model (Eq. (2)) for various experimental conditions are summarized in Figs. 3(a) and 4. The observed exponential dependence of steady states on q_B and the sigmoidal dependence on u_L can be explained by this model. In fact, quantitative agreements between our model and data are found everywhere except in very high magnetic fields where $q_B/h > 1000$ Hz, and in a moderate lattice depth ($4E_R \leq u_L \leq 6E_R$). The small discrepancy may be due to the limited resolution in TOF images, and the resulting larger uncertainties in measuring D and \mathbf{W} when quantum depletion does not saturate the first Brillouin zone. Heating induced by an additional magnetic coil in creating the very high q_B may also contribute to the discrepancy.

In conclusion, we have conducted the first experimental study on dynamics of lattice-trapped antiferromagnetic spinor BECs. Spin population oscillations and a lattice-tuned separatrix in phase space have been observed in every lattice where a substantial superfluid fraction exists. We have found that steady states of lattice-confined spinor BECs depend sigmoidally on u_L and exponentially on q_B , and undergo a polar-BA phase transition in a sufficiently deep lattice. We have also developed a phenomenological model that describes our data without adjustable parameters. While the underlying physics requires further study, this paper presents a few thought-provoking results on lattice-confined spinor BECs.

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* yingmei.liu@okstate.edu

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